Physics 201 Name KEY

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Exam 1 – Electrostatics

February 8, 2007

This is a closed book examination but you may refer to a 4"x5" note card with words of wisdom you have written on it during the exam. There is extra scratch paper available. Please explain your answers. Your explanation is worth 3/4 of the points on multiple-choice questions.

MSUM Mission Statement:
We develop knowledge, talent, and skills for a lifetime of learning, service, and citizenship.

1. [4 PTS] Given 10 identical capacitors, which of the following gives you the smallest effective capacitance in a circuit?
   a) Use just one capacitor.
   b) Connect all the capacitors in parallel.
   c) Connect all the capacitors in series.
   d) Connect half the capacitors in parallel which are then connected in series to the remaining capacitors.
   e) None of the above.

connect in series
C_{TOT} is reduced

$\frac{1}{C_{TOT}} = \sum \frac{1}{C_i}$ so $C_{TOT} = \frac{C_i}{N}$

2. [4 PTS] A capacitor has energy, E_i, stored in it. If the charge is reduced by half on the capacitor its stored energy, E_f,
   a) decreases $E_f=\frac{1}{4}E_i$.
   b) decreases $E_f=\frac{1}{2}E_i$.
   c) remains unchanged.
   d) increases $E_f=2E_i$.
   e) increase $E_f=4E_i$.

Energy stored in capacitor $\frac{1}{2}QV$
reducing charge – reduces voltage

$Q=CV$ so use $\frac{1}{2}\frac{Q^2}{C}$ $E_i = \frac{1}{2}\frac{Q^2}{C}$ $E_f = \frac{1}{2}\frac{(Q/2)^2}{C}$ $\frac{E_i}{E_f} = 4$

3. [4 PTS] By comparison with the force of gravity, the electrostatic force between two protons
   a) is repulsive and very much weaker.
   b) is also attractive and just about the same magnitude.
   c) is also attractive but very much stronger.
   d) is not measured in Newtons and hence cannot be compared.
   e) is repulsive and very much stronger.

← (+) (+) →

$F_q = \frac{k q_1 q_2}{r_{12}^2}$ $k = 9\times10^9 \frac{Nm^2}{C^2}$

$F_g = \frac{G m_1 m_2}{r_{12}^2}$ $G = 6.7\times10^{-11} \frac{Nm^2}{kg^2}$

Gravity is very weak

4. [4 PTS] When the center-to-center separation between a small positive sphere and a small negative sphere is decreased by 4 times, the electric force between them
   a) is reduced by 16.
   b) is reduced by 4.
   c) is reduced by 2.
   d) is increased by 2.
   e) is increased by 4.
   f) is increased by 16.
   g) none of these

$F_q = \frac{k q_1 q_2}{r^2}$ $F_i = \frac{k q_1 q_2}{r^2}$ $F_f = \frac{k q_1 q_2}{(r/4)^2}$ $\frac{F_i}{F_f} = \frac{1}{16}$

Force increases as you decrease the distance.

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5. [4 PTS] The volume of space between two parallel plates has a constant potential (voltage) everywhere in it. It follows that between these plates
   a) the electric field is proportional to $1/r^2$.
   b) the potential must be zero everywhere.
   c) the electric field is zero everywhere.
   d) the electric field is non-zero but constant.
   e) there is not enough information given to determine the electric field.

$E = \frac{-\partial(V)}{\partial r}$ so if $\Delta V = 0$ then $\vec{E} = 0$

6. [4 PTS] A large metal can has a charge deposited inside of it.
   a) The electric field in the metal is zero but the electric field inside the can is non-zero.
   b) The charge stays on the inside surface of the can.
   c) The charge moves to the outside surface of the can.
   d) The can is metal so the charge is neutralized.
   e) The potential inside the can is proportional to 1/r.

A conductor can not have an electric field inside it. The charge will move to the outside surface.

7. [4 PTS] An electron enters the region of space between two parallel plates held at 21 V and 96 V. The electron is initially traveling parallel to the plates. Which trajectory most closely resembles the electron's path?

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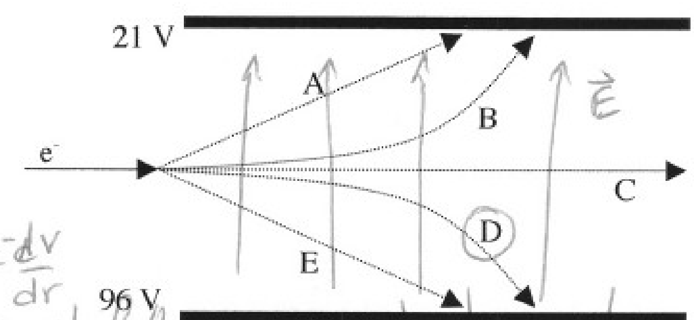

$a = F/m \quad F = qE \quad E = -\frac{dV}{dr}$

acceleration is constant so it follows curved trajectory

The next two problems can be done on the back of your exam or on additional paper.

8. [10 PTS] A ball of plasma has a charge density which decreases with radius. The ball has a radius $R_B = 5$ cm and charge density $\rho = 1.77 \times 10^{-8}/r \frac{C}{m^2}$.
   a) What is the electric field inside and outside the ball?
   b) What is the potential difference between $r_1 = 6$ cm and $r_2 = 11$ cm?

9. [10 PTS] You are working on a project and have built a vacuum chamber to smash fast moving alpha (α) particles into different crystals. The α-particles, 2 protons and 2 neutrons, are emitted by a radioactive substance that is located on a grounded plate (0 V). You would like to accelerate the α-particles to 1% the speed of light. Your vacuum chamber has three grids that are at 1.0 cm, 2.0 cm and 3.0 cm away from and parallel to the grounded plate. The first grid is held at -500 V while the second grid is held at -400 V. You can only change the voltage on the third grid. The speed of light is 3×10^8 m/s, the charge on a proton is 1.6×10^{-19} C and the mass of an α-particle is 6.6×10^{-27} kg. Assume the α-particles are initially at rest.
   a) Is this possible? If so what voltage would you need on the third grid?
   b) Indicate the strength of the electric field between the grids.

Volume Elements for: cartesian $dV = dxdydz$; cylindrical $dV = rdrd\theta dz$; spherical $dV = r^2 \sin\theta drd\theta d\phi$

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[8]

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$R_B = 0.05m$

$$\rho = \frac{1.77\times10^{-8}}{r}\frac{C}{m^2} = \frac{A}{r}\frac{C}{m^2} \qquad A = 1.77\times10^{-8}$$

Use Gauss' Law to find electric field

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{\sum q}{\epsilon_0} \qquad q_{enc} = \int \rho\, dV = \int \frac{A}{r}(4\pi r^2 dr)$$

Electric field is radial so // to surface

$$\Phi = E\,4\pi r^2$$

The volume integral reduces to integrating all the shells ($4\pi r^2$) of thickness (dr)

(A)

(a) Inside sphere

(r is inside sphere ($r < R_B$))

$$q_{enc} = \int_0^r \left(\frac{A}{r}\right)(4\pi r^2)dr = 4\pi A\int_0^r r\,dr = 4\pi A\,\frac{1}{2}r^2\Big|_0^r$$

$$= 4\pi A\,\frac{r^2}{2}$$

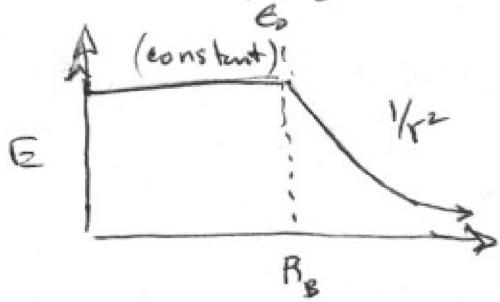
so $E\,4\pi r^2 = \frac{4\pi A}{\epsilon_0}\frac{r^2}{2}$ $\qquad \vec{E} = \frac{A}{2\epsilon_0}\hat{r}$ $\qquad$ constant $\vec{E}$ directed radially

(b) Outside sphere

(r is external to sphere $r > R_B$)

$$q_{TOT} = \int_0^{R_B}\left(\frac{A}{r}\right)(4\pi r^2)dr = 4\pi A\int_0^{R_B} r\,dr = 4\pi A\,\frac{1}{2}R_B^2$$

so $E\,4\pi r^2 = \frac{4\pi A}{\epsilon_0}\frac{1}{2}R_B^2$ $\qquad \vec{E} = \frac{A R_B^2}{2\epsilon_0}\frac{1}{r^2}\hat{r}$ $\qquad \frac{1}{r^2}$ directed radially



[8] continued

(B)

$$V = -\int_{r_1}^{r_2} \vec{E}\cdot d\vec{r}$$ this is outside so use $\vec{E} = \frac{AR_B^2}{2\epsilon_0}\frac{1}{r^2}\hat{r}$

$$= -\frac{AR_B^2}{2\epsilon_0}\int_{r_1}^{r_2}\frac{1}{r^2}\,dr = \frac{AR_B^2}{2\epsilon_0}\frac{1}{r}\Big|_{r_1}^{r_2} = \frac{AR_B^2}{2\epsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$ $r_1 = 0.06\,m$, $r_2 = 0.11\,m$

$\epsilon_0 = \frac{1}{4\pi k_0}$ so $\Delta V = \frac{A\,4\pi k_0 R_B^2}{2}\left(\frac{1}{0.11}-\frac{1}{0.06}\right) = -18.9\,V$

☐ units check (each step)

☐ ΔV is negative, E is from positive and you are moving away so this makes sense

$$V_{in} = -\int E\cdot dr$$

$$= -\int \frac{A}{2\epsilon_0}\,dr = -\frac{A}{2\epsilon_0}r + V_0 \qquad = -\frac{A}{2\epsilon_0}r + \frac{AR_B}{\epsilon_0}$$

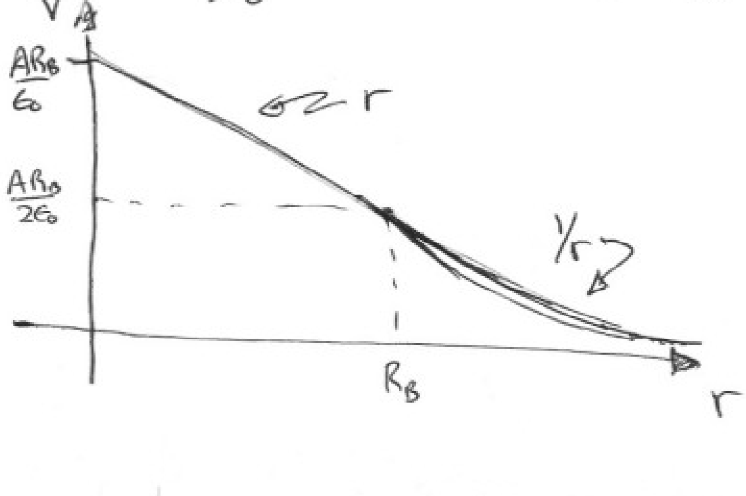
constant of integration

$$V_{out} = -\int \frac{A}{2\epsilon_0}R_B^2\frac{1}{r^2}\,dr = \frac{AR_B^2}{2\epsilon_0}\frac{1}{r} + \cancel{V_1} \to 0$$ $V(\infty) = 0$ so $V_1 = 0$

$V(R_B)$ has no discontinuity so

$$V_{in}(R_B) = V_{out}(R_B)$$

$$-\frac{A}{2\epsilon_0}R_B + V_0 = \frac{AR_B^2}{2\epsilon_0}\left(\frac{1}{R_B}\right) \qquad V_0 = \frac{AR_B}{\epsilon_0}$$



$V(0.06) = 41.7\,V$

$V(0.11) = 22.7\,V$

$\Delta V = 19\,V$

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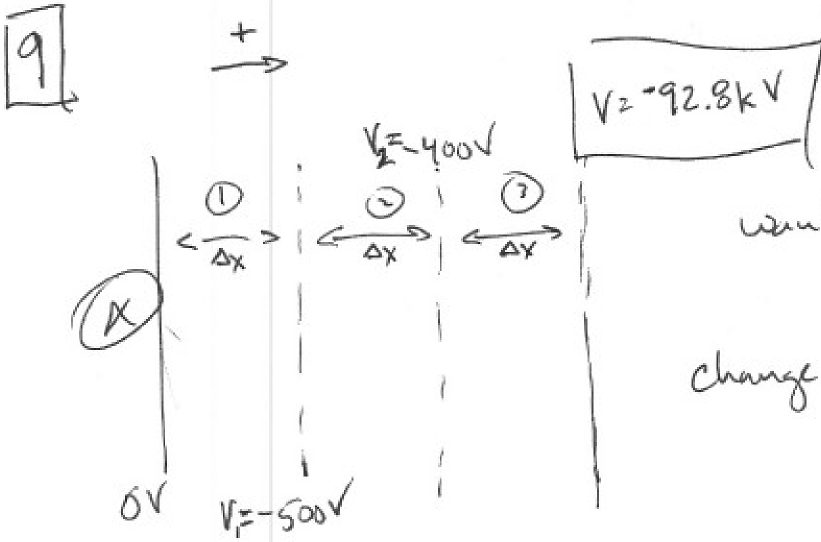

want $v = 0.01 \times 3\times10^8\ m/s = 3\times10^6\ m/s$

change in voltage is what matters

Use conservation of energy

$$\cancel{\tfrac{1}{2}m|\vec{v}_i|^2}^{\,0} + \cancel{qV_i}^{\,0} = \tfrac{1}{2}m|\vec{v}_f|^2 + qV_f$$

$V_i = 0V \qquad \vec{v}_i = 0\ m/s$

$V_f = ? \qquad \vec{v}_f = 3\times10^6\ m/s$

$$V_f = -\frac{m|v_f|^2}{2q}$$

$q = 2\times1.6\times10^{-19}\ C$ (2 protons)

$m = 6.6\times10^{-27}\ kg$

$$V_f = -\frac{6.6\times10^{-27}\,kg\,(3\times10^6\ m/s)^2}{2(2\cdot1.6\times10^{-19}\,C)}$$

(a) $V_f = -92.8\ kV$ — This is possible we had this kind of voltage on our VandeGraaf Generator

(b) $\vec{E} = -\dfrac{\partial V}{\partial r}$ in this case the electric field is uniform between the grids

$$E_1 = -\frac{\Delta V}{\Delta x} = \frac{+500V}{.01m} = 50\ kV/m$$

$\Delta x = 0.01\ cm$

$$E_2 = -\frac{(-400 - -500)V}{.01m} = \frac{-100V}{.01m} = -10\ kV/m$$

$$E_3 = -\frac{(-92.8kV - -400)V}{.01m} = \frac{+92.4kV}{.01m} = +9.24\ MV/m = 9240\ kV/m$$

check units ☑

$\vec{v}_f \uparrow \quad V \uparrow$ ← need more voltage to go faster

$q \uparrow \quad V \downarrow$ ← need less voltage to accelerate

$m \uparrow \quad V \uparrow$ ← need more voltage to move heavier particle